

# The Strange Sea Density and Charm Production in Deep Inelastic Charged Current Processes

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## Abstract

Charm production as related to the determination of the strange sea density in deep inelastic charged current processes is studied predominantly in the framework of the  $\overline{\text{MS}}$  fixed flavor factorization scheme. Perturbative stability within this formalism is demonstrated. The compatibility of recent next-to-leading order strange quark distributions with the available dimuon and  $F_2^{\nu N}$  data is investigated. It is shown that final conclusions concerning these distributions afford further analyses of presently available and/or forthcoming neutrino data.

Heavy quark production at high energy neutral current reactions was recently shown [1] to be optimally described within the framework of a fixed flavor (factorization) scheme (FFS) where, besides the gluon  $g$ , only the  $u$ ,  $d$  and  $s$  quarks are considered as partons and any heavy quark ( $c$ ,  $b$ , ...) contribution is calculated in fixed order  $\alpha_s$  perturbation theory. Within this framework the charged current production of a heavy quark pair such as  $t\bar{b}$  in  $\nu p \rightarrow \mu^- t\bar{b} X$  follows the same pattern [2] utilizing the relevant formulae for the underlying 'Wg fusion' subprocess  $W^+g \rightarrow t\bar{b}$ . Since both  $m_{t,b} \gg \Lambda_{QCD}$ , we do not encounter any mass singularities here and a treatment within the framework of the FFS is straightforward and unproblematic. This favorable situation changes, however, when considering the corresponding charm production process (e.g.  $W^+g \rightarrow c\bar{s}$ ) since the associated strange quark is taken as massless in the FFS and considered as a parton. In contrast to the former cases we encounter here a mixed situation which affords a careful treatment within the framework of the FFS. The leading order (LO) contribution for charm production in  $\nu N \rightarrow \mu^- c X$ ,  $N = (p+n)/2$ , comes from the basic  $\mathcal{O}(\alpha_s^0)$  subprocess  $W^+s' \rightarrow c$  where

$$s'_{\nu N} \equiv |V_{cs}|^2 s + |V_{cd}|^2 \frac{d+u}{2} \quad (1)$$

with  $|V_{cs}| = 0.9743$  and  $|V_{cd}| = 0.221$ . The  $W^+g \rightarrow c\bar{s}$  fusion process yields the essential part of the next-to-leading order (NLO) correction where the other part is due to the subprocess  $W^+s' \rightarrow gc$ . Both subprocesses possess a mass singularity associated with  $m_s = 0$  which is absorbed via dimensional regularization into the renormalized,  $Q^2$ -dependent, parton distribution  $s'$ . The remaining finite pieces of  $W^+g \rightarrow c\bar{s}$  and  $W^+s' \rightarrow gc$  then yield the genuine NLO correction to  $W^+s' \rightarrow c$  which will henceforth be considered in the now commonly adopted  $\overline{\text{MS}}$  factorization scheme. Denoting the contributions of the above subprocesses to the structure functions  $F_i(x, Q^2)$  by  $F_i^c(x, Q^2)$  and defining furthermore  $\mathcal{F}_1^c \equiv F_1^c$ ,  $\mathcal{F}_3^c \equiv F_3^c/2$ ,  $\mathcal{F}_2^c \equiv F_2^c/2\xi$  where  $\xi = x(1 + m_c^2/Q^2)$ , one obtains in NLO [3]

$$\begin{aligned} \mathcal{F}_i^c(x, Q^2) = & s'(\xi, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \left\{ \int_{\xi}^1 \frac{d\xi'}{\xi'} \left[ H_i^g(\xi', \mu^2, \lambda) s'(\frac{\xi}{\xi'}, \mu^2) \right. \right. \\ & \left. \left. + H_i^q(\xi', \mu^2, \lambda) g(\frac{\xi}{\xi'}, \mu^2) \right] \right\} . \end{aligned} \quad (2)$$

Here  $\lambda \equiv Q^2/(Q^2 + m_c^2)$  and the  $H_i^{q,g}$  are given, up to minor modifications specified in the Appendix, in ref. [3]. The specific choice for the factorization scale  $\mu$  will be studied below. The inclusive cross section in terms of the  $F_i(x, Q^2)$  is given by

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2/M_W^2)^2} \left[ (1-y)F_2^{\nu(\bar{\nu})} + y^2 x F_1^{\nu(\bar{\nu})} \pm y(1 - \frac{y}{2})x F_3^{\nu(\bar{\nu})} \right] . \quad (3)$$

To study the size of the NLO corrections we shall utilize the LO and NLO parton distributions of [4] which are already conceived in the FFS being furthermore  $\overline{\text{MS}}$  distributions in the NLO. In addition we shall also employ the LO and NLO( $\overline{\text{MS}}$ ) CTEQ3 [5] and the NLO( $\overline{\text{MS}}$ ) MRS(A) [6] parton densities which refer to the 'variable flavor' scheme where intrinsic charm densities are purely radiatively generated using the ordinary massless evolution equations, starting at  $Q = m_c$ . For definiteness we show in fig. 1 the quantity

$$\xi s(\xi, Q^2)_{eff} \equiv \frac{1}{2} \frac{\pi(1 + Q^2/M_W^2)^2}{G_F^2 M_N E_\nu} |V_{cs}|^{-2} \frac{d^2\sigma^{(c\bar{s})}}{dx dy} \quad (4)$$

which has been also studied experimentally [7] and where the superscript  $c\bar{s}$  refers just to the CKM non-suppressed ( $V_{cs}$ ) component of  $s'$  in eqs. (1–3). Note that in LO the cross section in (4) reduces to

$$\xi s(\xi, Q^2)_{eff} = (1 - \frac{m_c^2}{2M_N E_\nu \xi}) \xi s(\xi, \mu^2) + \mathcal{O}(\alpha_s) . \quad (4')$$

As can be seen in fig. 1 the NLO corrections to the LO results are reasonably small and, in particular, do not afford a drastic change of  $s(x, Q^2)$  when passing from the LO to the NLO analysis.

This contrasts with the conclusions of the CCFR group [7] whose NLO  $s(x, Q^2)$  is almost twice as large as compared to their previous [8] LO  $s(x, Q^2)$ . The analysis of the CCFR group is based on the NLO formalism of [9] which is not strictly equivalent to our NLO( $\overline{\text{MS}}$ ) FFS formalism but still is expected to yield quite similar results. The enhancement of the NLO  $s(x, Q^2)$  in [7] can therefore not be attributed to the different formalism itself [10] but rather to its inconsistent application. In the formalism of ref. [9] one considers the  $W^+g \rightarrow c\bar{s}$  contribution with  $m_s \neq 0$ , i. e. employs a finite mass

regularization and subtracts from it that (collinear) part which is already contained in the renormalized,  $Q^2$ -dependent  $s(x, Q^2)$ . The CCFR group applied their acceptance corrections to the full contribution from  $W^+g \rightarrow c\bar{s}$ , which corresponds effectively to a multiplication with an acceptance correction factor  $\mathcal{A} = 0.6 \pm 0.1$ , while inconsistently keeping the subtraction term in its full original (acceptance uncorrected) strength [11]. This latter subtraction term is given, relative to  $\xi s(\xi, \mu^2)$ , by

$$\text{SUB} \equiv \frac{\alpha_s(\mu^2)}{2\pi} \ln \frac{\mu^2}{m_s^2} \int_{\xi}^1 \frac{dz}{z} g(z, \mu^2) P_{qg}^{(0)} \left( \frac{\xi}{z} \right) \quad (5)$$

using [11]  $m_s = 0.2$  GeV. In fig. 2 we compare the result obtained in this manner with the one where also the subtraction term [9] in (5) was consistently multiplied by the same acceptance factor  $\mathcal{A}$ . The result corresponding to the acceptance uncorrected subtraction term (dashed curve) clearly demonstrates that SUB alone [eq. (5)] represents too strong a suppression of the mass singularity component in  $\text{LO} + \mathcal{A} * \text{NLO}$  and that the correct result (solid curve) in fig. 2 is almost a factor of 2 larger in the small- $x$  region. This implies that instead of  $s_{\text{NLO}} \simeq 2 s_{\text{LO}}$  for  $x \lesssim 0.1$ , as inferred by CCFR [7] and used for our analysis in fig. 2, one rather needs a smaller  $s_{\text{NLO}}$ , i. e. closer in size to  $s_{\text{LO}}$ , in order to reduce the solid curve in fig. 2 and to bring it closer to experiment. Here we have chosen [7] a factorization scale  $\mu = 2 p_T^{\text{max}} = \Delta(W^2, m_c^2, M_N^2)/W$ , where  $p_T^{\text{max}}$  is the maximum available transverse momentum of the final state charm quark; the results in fig. 2 remain practically unaltered with the alternative choice  $\mu^2 = Q^2 + m_c^2$ .

A further feature emanating from the fits in [7] was  $m_c^{\text{NLO}} \simeq 1.7$  GeV as compared to the previous [8]  $m_c^{\text{LO}} \simeq 1.3$  GeV which further suppressed the NLO cross section and demanded the unusual, even more enhanced NLO  $s(x, Q^2)$ . A consistent treatment of the acceptance correction would most probably also lower the fitted  $m_c^{\text{NLO}}$  down to a more reasonable  $m_c^{\text{NLO}} \simeq 1.5$  GeV and bring the NLO  $s(x, Q^2)$  close to the LO  $s(x, Q^2)$ .

Our conclusions concerning the strange quark distributions of [4, 5] are that they agree in LO with [8] and are not refuted in NLO by the analysis in [7]. Furthermore due to the perturbative stability demonstrated in fig. 1 we expect the NLO strange quark distributions of [4–6] to lie in the correct ball park. For a final conclusion concerning these

matters, a reanalysis of presently available dimuon neutrino data is obviously mandatory!

It is also interesting to check the above statements by an independent quantitative test sensitive to  $s(x, Q^2)$  such as for example the combination  $\frac{5}{6}F_2^{\nu N} - 3F_2^{\mu N}$  which is given approximately by

$$\frac{5}{6} F_2^{\nu N}(x, Q^2) - 3 F_2^{\mu N}(x, Q^2) \simeq xs(x, Q^2) \quad (6)$$

where the charm contributions, the  $m_c^2/Q^2$  corrections and the NLO  $q$ - and  $g$ - induced contributions are rather small and, furthermore,  $|V_{cs}|^2 \simeq 1$  and  $|V_{cd}|^2 \simeq 0$ . In fig. 3a we compare various LO and NLO results for  $xs(x, Q^2)$  in eq. (6) with the published [12] and more recent but preliminary [13]  $\bar{\nu}N$  data and the NMC (deuteron)  $\mu N$  data [14]. It should be kept in mind that the neutrino data refer to a Fe-target and are therefore very sensitive to nuclear (EMC) corrections in the small- $x$  region: Only the preliminary (unpublished) neutrino data [13], which are larger than the published ones [12] in the small- $x$  region, disagree with the approximate predictions, eq. (6), in fig. 3a. That this latter approximation is indeed sufficiently accurate is demonstrated in fig. 3b where  $xs(x, Q^2)$  is compared with the full NLO result for  $\frac{5}{6}F_2^{\nu N} - 3F_2^{\mu N}$  which has to be calculated in the following way (note that  $F_2^{\nu N}$  always refers to an average over  $\nu$  and  $\bar{\nu}$ ). In the FFS we have

$$F_2^{(\bar{\nu})N}(x, Q^2) = x \sum_{q=u,d} \left\{ (q' + \bar{q}')(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left[ ((q' + \bar{q}') * C_2^q)(x, Q^2) + 2 (g * C_2^g)(x, Q^2) \right] \right\} + 2 \xi \mathcal{F}_2^{c(-)}(x, Q^2) \quad (7)$$

with  $q' = \frac{1}{2}(1 + |V_{ud}|^2)q + \frac{1}{2}|V_{us}|^2s$ , using  $|V_{ud}|^2 + |V_{us}|^2 = 1$  with  $|V_{ud}|^2 = 0.9743$ , and where  $\mathcal{F}_2^c$  is given in eq. (2) with the replacement  $s' \rightarrow \frac{1}{2}(s' + \bar{s}')$ . The massless coefficient functions  $C_2^{q,g}$  are standard, see e. g. ref. [4], and the convolutions are defined by

$$(q * C)(x, Q^2) = \int_x^1 \frac{dz}{z} q(z, Q^2) C\left(\frac{x}{z}\right) .$$

The well known expression for  $F_2^{\mu N}$  [4] is appropriately modified for an isoscalar target, with the charm contribution  $F_2^{c\bar{c}}$  calculated according to the  $\gamma^*g \rightarrow c\bar{c}$  fusion process

etc. [1] as described, for example, in [4]. In the 'variable flavor' scheme [5, 6], where intrinsic charm densities  $c(x, Q^2)$  are generated radiatively by the ordinary massless evolution equations, we have

$$\frac{1}{x} \left( \frac{5}{6} F_2^{\nu N} - 3 F_2^{\mu N} \right) = (s - c)(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} [(s - c) * C_2^q](x, Q^2) \quad . \quad (8)$$

In view of the preliminary and contradicting nature of the nuclear-shadowing corrected CCFR data for  $F_2^{(\nu)N}$  used in fig. 3, a decision concerning the (dis)agreement with theoretical QCD predictions must obviously be postponed. According to our results in figs. 1 and 2, implying strongly that  $s_{\text{NLO}}$  is similar in size to  $s_{\text{LO}}$ , and the ones in fig. 3 which imply that the inclusion of the finite part of  $W^+g \rightarrow c\bar{s}$  and the corresponding photon induced  $\gamma^*g \rightarrow c\bar{c}$  in conjunction with present NLO strange quark densities [4–6] do not change significantly the simple LO results, the theoretical predictions are rather constrained and unique. Furthermore, the results in fig. 3 again support our previous conclusions [1] concerning the perturbative stability [3, 15] of the charm production rate as calculated in perturbative fixed order  $\alpha_s$ , i. e. in the FFS. A similar analysis was carried out in [16] where different conclusions concerning the magnitude of the gluon induced contributions are presented: These results are almost a factor of two larger than the full NLO results at  $x = 10^{-2}$  in fig. 3b since a factor of two error seems (due to the lack of explicit formulae in [16] it is not possible to trace its exact origin) to be present in the calculation of the  $W^+g \rightarrow c\bar{s}$  contribution. Therefore, if the enhanced preliminary  $\nu N$  data at  $x \lesssim 0.1$  as shown in fig. 3 are confirmed, the discrepancy between these data and the rather solid and unique theoretical results, taking into account the rather well understood dimuon data as well, will constitute a major problem which cannot be solved within our present understanding of the so far successful perturbative QCD.

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## Appendix

The fermionic NLO coefficient functions  $H_i^q$  for heavy quark (charm) production in eq. (2), calculated from the subprocess  $W^+s \rightarrow gc$ , are given by

$$H_i^q(z, \mu^2, \lambda) = \left[ P_{qq}^{(0)}(z) \ln \frac{Q^2 + m_c^2}{\mu^2} + h_i^q(z, \lambda) \right] \quad (\text{A1})$$

where  $P_{qq}^{(0)}(z) = \frac{4}{3} \left( \frac{1+z^2}{1-z} \right)_+$  and

$$h_i^q(z, \lambda) = \frac{4}{3} \left\{ h^q + A_i \delta(1-z) + B_{1,i} \frac{1}{(1-z)_+} + B_{2,i} \frac{1}{(1-\lambda z)_+} + B_{3,i} \left[ \frac{1-z}{(1-\lambda z)^2} \right]_+ \right\} \quad (\text{A2})$$

with

$$h^q = - \left( 4 + \frac{1}{2\lambda} + \frac{\pi^3}{3} + \frac{1+3\lambda}{2\lambda} K_A \right) \delta(1-z) - \frac{(1+z^2) \ln z}{1-z} + (1+z^2) \left[ \frac{2 \ln(1-z) - \ln(1-\lambda z)}{1-z} \right]_+ \quad (\text{A3})$$

and

$$K_A = \frac{1}{\lambda} (1-\lambda) \ln(1-\lambda) \quad . \quad (\text{A4})$$

The coefficients in (A2) for  $i = 1, 2, 3$  are given in Table 1 where a misprint in [3] concerning  $A_2$  was corrected.

Table 1. Coefficients for the expansion of  $h_i^q$  in (A2)

| $i$ | $A_i$ | $B_{1,i}$                | $B_{2,i}$             | $B_{3,i}$     |
|-----|-------|--------------------------|-----------------------|---------------|
| 1   | 0     | $1 - 4z + z^2$           | $z - z^2$             | $\frac{1}{2}$ |
| 2   | $K_A$ | $2 - 2z^2 - \frac{2}{z}$ | $\frac{2}{z} - 1 - z$ | $\frac{1}{2}$ |
| 3   | 0     | $-1 - z^2$               | $1 - z$               | $\frac{1}{2}$ |



The gluonic NLO coefficient functions  $H_i^g$  for heavy quark (charm) production in eq. (2), as calculated from the subprocess  $W^+g \rightarrow c\bar{s}$ , are given by

$$H_{i=1,2,3}^g(z, \mu^2, \lambda) = \left[ P_{qg}^{(0)}(z) \left( \pm L_\lambda + \ln \frac{Q^2 + m_c^2}{\mu^2} \right) + h_i^g(z, \lambda) \right] \quad (\text{A5})$$

where  $P_{qg}^{(0)}(z) = \frac{1}{2} [z^2 + (1-z)^2]$ ,  $L_\lambda = \ln \frac{1-\lambda z}{(1-\lambda)z}$  and

$$h_i^g(z, \lambda) = C_0 + C_{1,i} z(1-z) + C_{2,i} + (1-\lambda) z L_\lambda (C_{3,i} + \lambda z C_{4,i}) \quad (\text{A6})$$

with

$$C_0 = P_{qg}^{(0)}(z) [2 \ln(1-z) - \ln(1-\lambda z) - \ln z] \quad . \quad (\text{A7})$$

The coefficients  $C_{k,i}$  are given in Table 2 and differ from those in [3] where the older convention [17] has been adopted of counting the gluonic helicity states in  $D = 4$  rather than in  $D = 4 + 2 \varepsilon$  dimensions. The latter convention [18] is the one chosen to define all modern NLO parton distributions.

Table 2. Coefficients for the expansion of  $h_i^g$  in (A6)

| $i$ | $C_{1,i}$                                  | $C_{2,i}$                              | $C_{3,i}$  | $C_{4,i}$    |
|-----|--|--|------------|--------------|
| 1   | $4 - 4(1-\lambda)$                         | $\frac{(1-\lambda)z}{1-\lambda z} - 1$ | 2          | -4           |
| 2   | $\frac{8-18(1-\lambda)}{+12(1-\lambda)^2}$ | $\frac{1-\lambda}{1-\lambda z} - 1$    | $6\lambda$ | $-12\lambda$ |
| 3   | $2(1-\lambda)$                             | 0                                      | $-2(1-z)$  | 2            |

Note that in the limit  $\lambda \rightarrow 1$  ( $m_c \rightarrow 0$ ) the  $H_i^{q,g}$  reduce, apart from the obvious collinear logs, to the massless  $\overline{\text{MS}}$  coefficient functions  $C_i^{q,g}$  [4, 18].

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## Figure Captions

**Fig. 1** LO and NLO predictions for  $\xi_{seff}$  defined in eq. (4), using the GRV [4] and CTEQ3 [5] parton densities. The dotted curves refer to using just the NLO strange quark contribution in eq. (2) with all  $\mathcal{O}(\alpha_s)$  terms neglected. Thus the differences between the dashed and dotted curves illustrate the differences between the LO and NLO strange sea densities, respectively. The values of  $Q^2$  vary between 2.4 to 43.9 GeV<sup>2</sup> according to the experimental averages [8] for  $0.015 \leq x \leq 0.35$  and  $E_\nu = 192$  GeV [7] has been used.

**Fig. 2** NLO results using the NLO strange sea density of CCFR [7]. The subtraction term (SUB) is defined in (5) and an acceptance correction factor  $\mathcal{A} = 0.6$  has been used [11]. The analysis was performed with the original subroutines/matrix elements of CCFR [11]; if the charged current structure functions of GGR [2] are used instead, the results are similar. The shaded area refers to the CCFR 'data' [8], calculated according to (4'), where the CKM suppressed contribution in (1) has been subtracted from the measured full cross section by assuming specific up and down quark densities [7, 11]. The dashed curve corresponds to the original CCFR fit analysis [7, 11].

**Fig. 3** (a): LO and NLO GRV [4] and MRS(A) [6] predictions for  $xs(x, Q^2)$  which approximates  $\frac{5}{6}F_2^{\nu N} - 3F_2^{\mu N}$  in eq. (6). (b): Full NLO result for  $\frac{5}{6}F_2^{\nu N} - 3F_2^{\mu N}$  in the FFS (dashed dotted curve) using eq. (7), and the short-dashed curve shows the corresponding result with the  $W^+g \rightarrow c\bar{s}$  contribution turned off. The factorization scale chosen is  $\mu^2 = Q^2 + m_c^2$ . The solid curve for  $xs$ , being the same as in (a), is shown for comparison. The full NLO MRS(A) result in the 'variable flavor' scheme is based on eq. (8). Both CCFR [12] (circles) and preliminary CCFR [13] (squares)  $\nu N$  (Fe-target) data are corrected for nuclear shadowing effects, whereas the NMC  $\mu N$  data [14] have been obtained from a deuterium target.

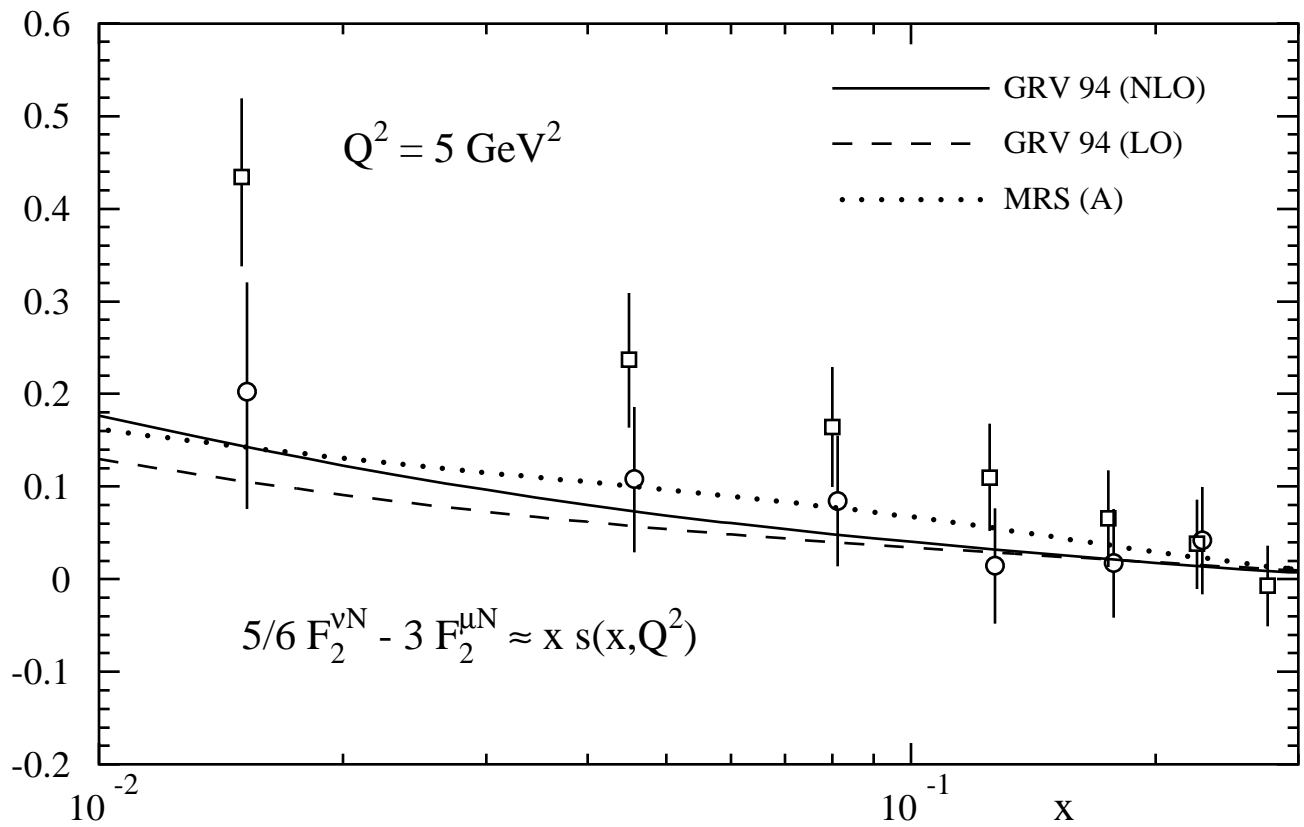


Fig. 3a

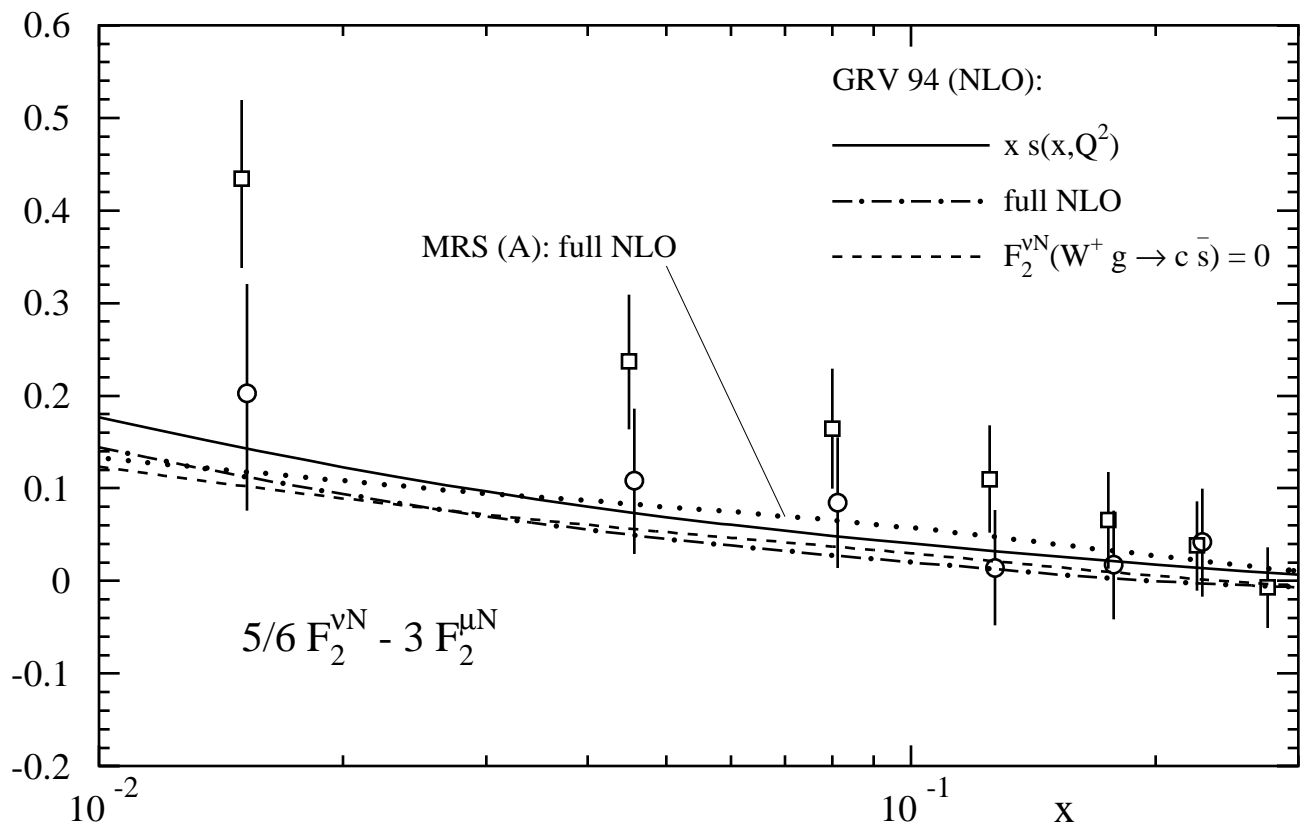


Fig. 3b

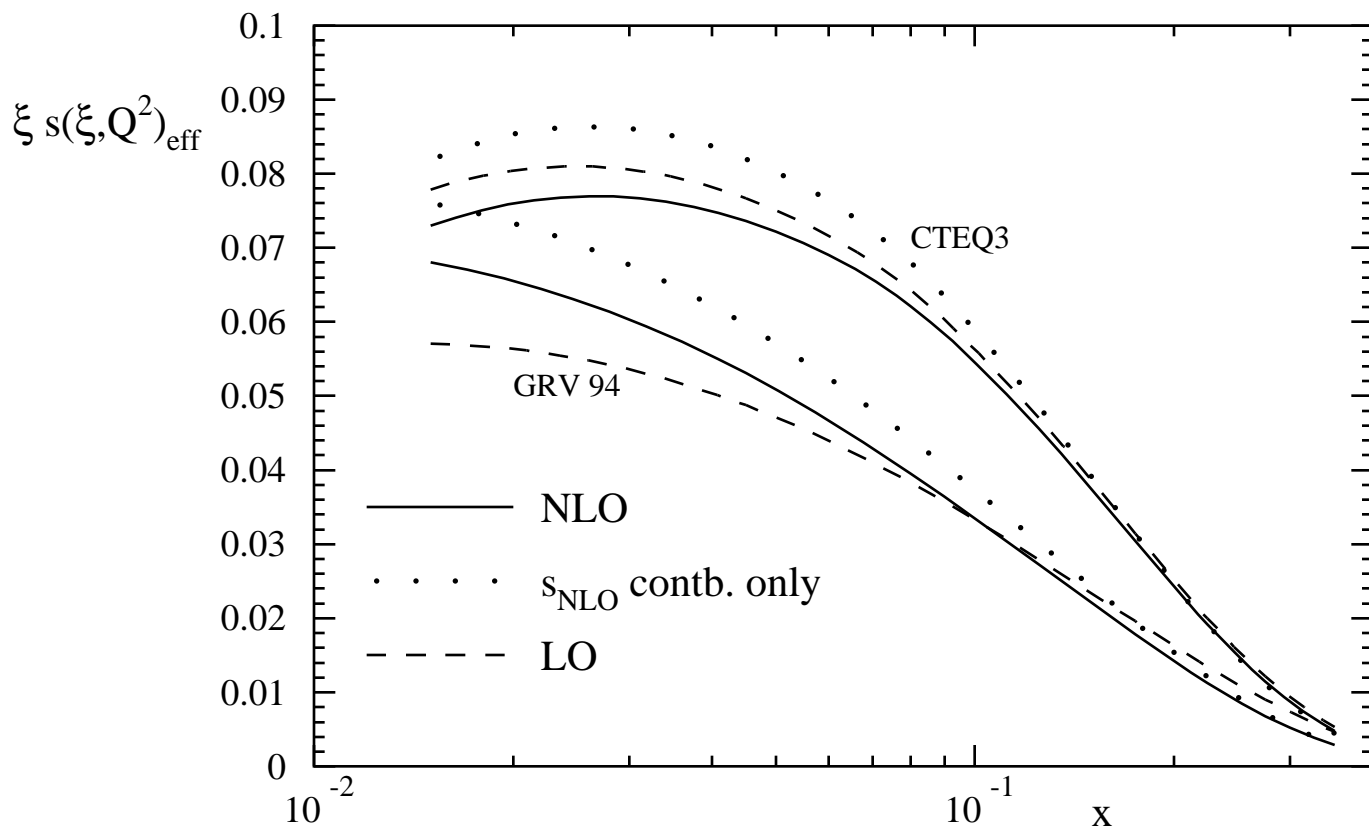


Fig. 1

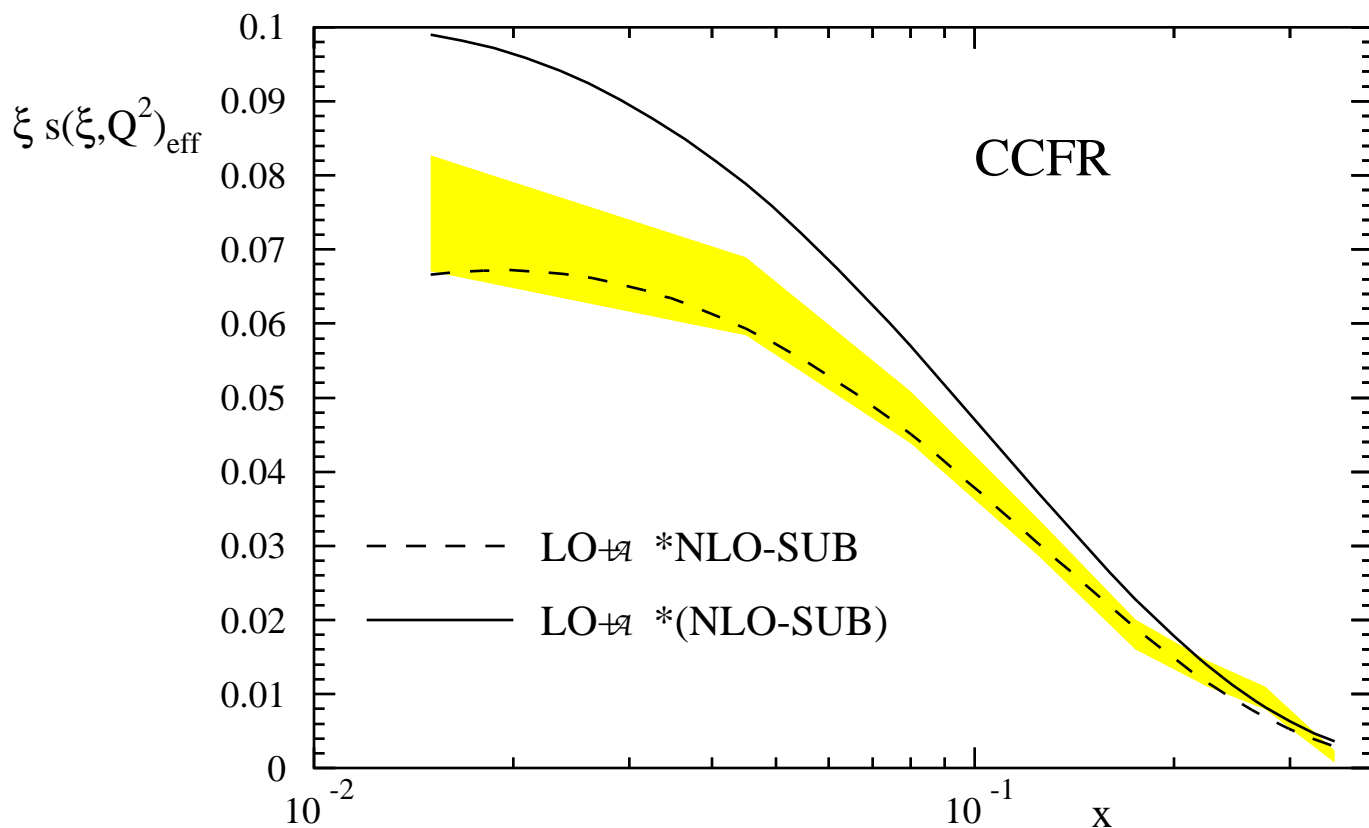


Fig. 2